

# Markscheme

**May 2023**

**Mathematics:  
applications and interpretation**

**Standard level**

**Paper 1**

20 pages

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## Instructions to Examiners

### Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

### Using the markscheme

#### 1 General

Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.

#### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award <b>A1</b> for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award <b>A0</b> for the final mark (and full <b>FT</b> is available in subsequent parts)

### 3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

### 4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

**For example:** following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

## 5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

## 6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

## 7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

## 8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a “correct” level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come “from the use of 3 sf values”.

**Simplification of final answers:** Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example,  $\sqrt{\frac{25}{4}}$  should be written as  $\frac{5}{2}$ .

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example,  $\frac{10}{4}$  may be left in this form or written as  $\frac{5}{2}$ . However,  $\frac{10}{5}$  should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g.  $4e^{2x} \times e^{3x}$  should be simplified to  $4e^{5x}$ , and  $4e^{2x} \times e^{3x} - e^{4x} \times e^x$  should be simplified to  $3e^{5x}$ . Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so  $x(x+1)$  and  $x^2 + x$  are both acceptable.

**Please note:** intermediate **A** marks do NOT need to be simplified.

## 9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

## 10. Presentation of candidate work

**Crossed out work:** If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

**More than one solution:** Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) attempt to substitute into percentage error formula **(M1)**
- $$\left| \frac{53632000 - 55625000}{55625000} \right| \times 100$$
- 3.58 (%) (3.58292...(%)) **A1**

**Note:** Award **(M1)A0** for a final answer of  $-3.58(\%)$  or  $0.0358$ .

**[2 marks]**

- (b) (i) 278 000 000 **A1**
- (ii)  $2.78 \times 10^8$  **A1A1**

**Note:** Award **A1** for correct mantissa, consistent with their answer in part (b)(i).  
Award **A1** for a correct exponent, consistent with their answer in part (b)(i).  
Award **A0A0** for answers such as  $27.8 \times 10^7$ .

**[3 marks]**

**Total [5 marks]**



2. (a) **METHOD 1 (use of financial app in GDC)**

$N = 5$                       **OR**                       $N = 20$   
 $I\% = 1.2$                        $I\% = 1.2$   
 $PV = \pm 520$                        $PV = \pm 520$   
 $P/Y = 1$                        $P/Y = 4$   
 $C/Y = 4$                        $C/Y = 4$

**(M1)(A1)**

**Note:** Award **M1** for evidence of using the financial app on the calculator, **A1** for all correct entries.

(\$) 552.11

**A1**

**Note:** Award at most **(M1)(A1)A0** if correct answer is not given to two decimal places.

**METHOD 2 (use of formula)**

attempt to substitute into compound interest formula

**(M1)**

$$520 \times \left( 1 + \frac{1.2}{100 \times 4} \right)^{5 \times 4}$$

**(A1)**

(\$) 552.11

**A1**

**Note:** Award at most **(M1)(A1)A0** if correct answer is not given to two decimal places.

**[3 marks]**

(b) **EITHER**

$N = 5$   
 $I\% = 43.5$  (43.4772...(%))  
 $PV = \pm 520$   
 $FV = \mp 30$

**(M1)(A1)A1**

**Note:** Award **M1** for evidence of using the finance app on the calculator, **A1** for all correct entries, **A1** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

**OR**

$$30 = 520 \left( 1 - \frac{r}{100} \right)^5 \quad (\text{or equivalent})$$

**(M1)(A1)**

$(r =) 43.5\%$  (43.477...%)

**A1**

**Note:** Award **M1** for using the compound interest formula, **A1** for correct substitutions and for equating to 30, **A1** for correct final answer. Accept  $(r =) -43.5\%$ . Award **M1A1A0** for a final answer of 56.5%.

**[3 marks]**

**Total [6 marks]**

3. (a) (i) 38 (s) **A1**  
(ii) 32 (s) **A1**  
(iii) 42 (s) **A1**  
(iv) 10 (s) **A1**

**Note:** Accept a tolerance of  $\pm 0.5$  for parts (a)(i)-(iii).

**[4 marks]**

- (b) 1.5 x IQR **(M1)**  
(32 - 1.5 × 10 =) 17 (s) **A1**  
14 < 17, therefore it is an outlier **R1**

**Note:** Do not award the **R1** unless an explicit comparison of 14 and their 17 is seen.  
e.g. 14 < 17  
14 is outside the interval [17, 57].

**[3 marks]**

**Total [7 marks]**

4. (a)

Athlete	A	B	C	D	E	F	G	H
Age rank	7	6	3	5	4	2	8	1
Time rank	3.5	2	3.5	6	7	8	1	5

**A1A1**

**Note:** Award **A1** for each correct row.

**[2 marks]**

(b)  $r_s = -0.671$  ( $-0.670670\dots$ )

**A2**

**Note:** Only follow through from an incorrect table provided the ranks are all between 1 and 8.  
Award **A1** for  $-0.67$  **OR** for the omission of the negative sign, e.g.  $0.671$  ( $0.670670\dots$ ) or  $0.67$

**[2 marks]**

(c) (A value of  $r_s = -0.671$ ) indicates a negative correlation between a person's age and the best time they take to run 100m.

**R1**

**Note:** Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run".  
Answer must be in context.

**[1 mark]**

(d) Award **R1** for any sensible reason:

**R1**

The correlation, such that it is, is unlikely to be linear for this type of data.  
Spearman's CC is less sensitive to outliers  
Sung-Jin is not sure the data is drawn from a bivariate normal distribution  
There are outliers/extreme data  
Same time for two athletes with significantly different ages

**[1 mark]**

**Total [6 marks]**

5. (a)  $34 + p$  **A1**  
[1 mark]
- (b) attempt to substitute into the mean formula, equating to 4.5 **(M1)**  

$$\frac{1 \times 1 + 2 \times 4 + \dots + 5 \times p + 6 \times 9 + 7 \times 4}{34 + p} = 4.5$$
 **A1**  
 $(p =) 10$  **A1**

**Note:** Do not award the final **A1** if final answer is not an integer.  
Award **(M1)A0A1** for an unsupported answer of  $(p =) 10$ .

[3 marks]  
Total [4 marks]

6. (a)  $0.565 \left( 0.564655\dots, \frac{131}{232}, 56.4655\dots\% \right)$  **A1A1**

**Note:** Award **A1** for correct numerator, **A1** for correct denominator.

[2 marks]

- (b) 11.0 (11.0212...) **A2**

**Note:** Award **A1** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

- (c) **EITHER**  
 $11.0 > 9.488$  ( $11.0212\dots > 9.488$ ) **R1**

**OR**  
 $0.0263 < 0.05$  ( $0.0263264\dots < 0.05$ ) **R1**

**THEN**

**EITHER**  
 (there is significant evidence to) reject  $H_0$  **A1**

**OR**  
 (there is significant evidence that) the (food) quality and the type of meal are not independent **A1**

**Note:** Do not award **R0A1**.  
Award **R1** for  $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$ , provided the calculated value is explicitly seen in part (b).  
  
Accept " $p$ -value < significance level" provided their  $p$ -value is seen and their  $p$ -value is between 0 and 1.

[2 marks]  
Total [6 marks]

7. (a) attempt to find gradient of CD **(M1)**  
 gradient of CD =  $\frac{1}{3}$  therefore perpendicular gradient CD =  $-3$  **A1**  
 $y - 1.5 = -3(x - 5.5)$  **OR**  $1.5 = -3(5.5) + c$  **M1**

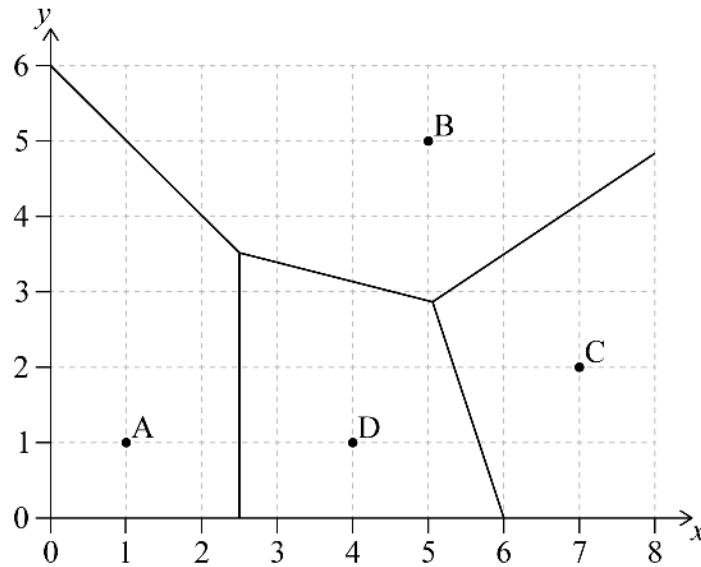
**Note:** Award **M1** for substituting the gradient and midpoint into equation of line, provided further work is seen leading to a correct answer.

$y = -3x + 18$

**AG**

**[3 marks]**

(b)



perpendicular bisector AD: a vertical line with  $x$  intercept 2.5

**A1**

**Note:** The perpendicular bisector should not go beyond the intersection point (should not enter site B).

**[1 mark]**

- (c) attempt to solve simultaneous equations:  $3y = 2x - 1.5$  and  $y = -3x + 18$  **(M1)**  
 $(5.05, 2.86)$   $((5.04545\dots, 2.86363\dots))$  **A1**

**Note:** Accept  $x = 5.05$  ( $5.04545\dots$ ),  $y = 2.86$  ( $2.86363\dots$ ) in place of coordinates.  
 Accept  $(5.05, 2.87)$  and  $(5.05, 2.85)$  for using their 3 sf or 4 sf  $x$ -value to find  $y$  from any of the two equations.

**[2 marks]**

**Total [6 marks]**

8. (a) **EITHER**

$$\frac{4}{3}\pi(3.4)^3$$

**(A1)**

multiplying their volume by  $\frac{4}{5}$

**(M1)**

**OR**

$$\frac{4}{3}\pi(3.4)^3$$

**(A1)**

Subtracting  $\frac{1}{5}$  of their volume

**(M1)**

$$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5} \times \frac{4}{3}\pi(3.4)^3\right)$$

**Note:** The **M1** can be awarded for a final answer of 32.9272... seen without working.

**THEN**

$$132 \text{ cm}^3 \text{ (131.708... cm}^3\text{)}$$

**A1**

**[3 marks]**

(b)  $\pi \times 3 \times 11$

**(A1)**

$$103.672... \text{ (cm}^2\text{)} \quad \text{OR} \quad 33\pi \text{ (cm}^2\text{)}$$

$$104 \text{ (cm}^2\text{)}$$

**A1**

**[2 marks]**

**Total [5 marks]**

9. (a)  $X \sim N(4, 0.25^2)$

**EITHER**

correct probability expression

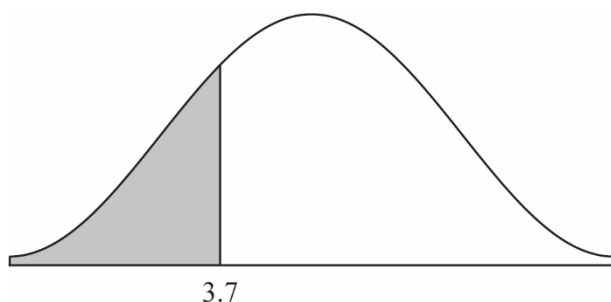
**(M1)**

$P(X < 3.7)$

**Note:** Accept a weak or strict inequality, and any label instead of  $X$ , e.g. length or  $L$ .

**OR**

normal curve with vertical line, left of mean, labelled 3.7, and shaded region **(M1)**



**THEN**

0.115 (0.115069..., 11.5%)

**A1**

**Note:** Award **M1A0** for 0.12 if no previous working.

**[2 marks]**

(b) **EITHER**

Correct probability expression

**(M1)**

$(P(X < k) = 0.7 \text{ OR } P(X > k) = 0.3)$

**Note:** Accept a weak or strict inequality, and any label instead of  $X$  e.g., length or  $L$ .

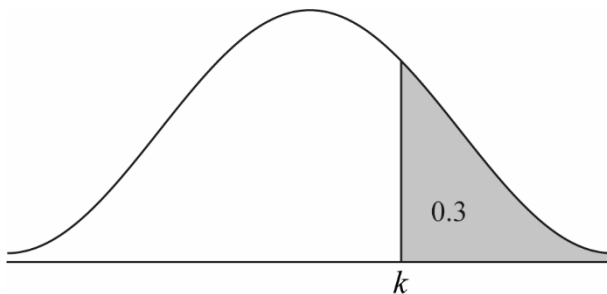
*continued...*

Question 9 continued

**OR**

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7

**(M1)**



**THEN**

$(k =) 4.13 (4.13110\dots)$

**A1**

**Note:** Award **M1A0** for 4.1 if no previous working.

**[2 marks]**

(c) **EITHER**

correct probability equation

**(M1)**

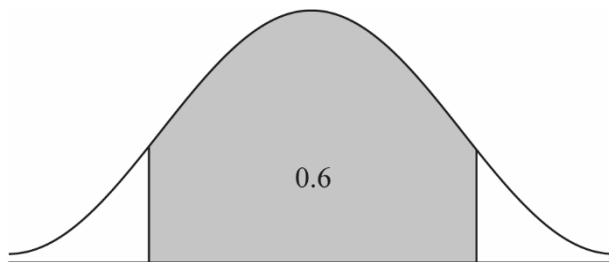
$P(\text{length} < 4 + m) = 0.8$  **OR**  $P(\text{length} < 4 - m) = 0.2$

**Note:** Accept any letter instead of "length" e.g.,  $X$  or  $L$ .

**OR**

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8

**(M1)**



**THEN**

0.210 (0.210405...)

**A1**

**Note:** Award **(M1)A0** for an answer of 3.7895 or 4.2105 seen without working.

**[2 marks]**

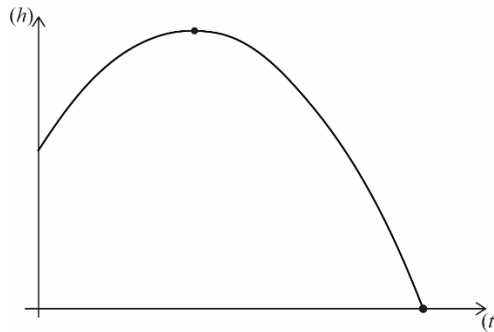
**Total [6 marks]**



10. (a) **METHOD 1**

correct sketch with some indication of maximum point

**(M1)**



$$0.921 \text{ (seconds)} \left( 0.921052\dots, \frac{35}{38} \right)$$

**A1**

**METHOD 2**

correct substitution into equation for line of symmetry

**(M1)**

$$(t =) -\frac{8.75}{2 \times -4.75}$$

$$0.921 \text{ (seconds)} \left( 0.921052\dots, \frac{35}{38} \right)$$

**A1**

**METHOD 3**

equating the correct derivative to 0

**(M1)**

$$-9.5t + 8.75 = 0$$

$$0.921 \text{ (seconds)} \left( 0.921052\dots, \frac{35}{38} \right)$$

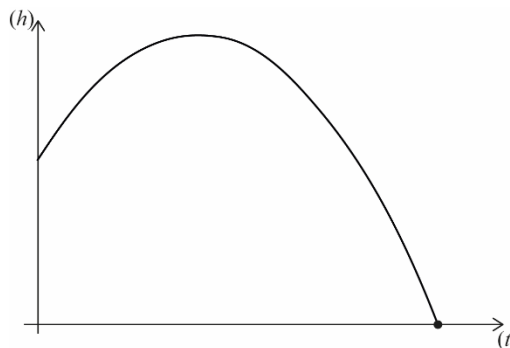
**A1**

**Note:** Award **M1A0** for a final answer of 0.92 seen with no working.

**[2 marks]**

(b) **METHOD 1**

correct sketch with some indication of x-intercept



**(M1)**

**Note:** May be seen in part (a).

$$2 \text{ (seconds)}$$

**A1**

*continued...*

Question 10 continued

**METHOD 2**

setting the equation to zero

**(M1)**

$$-4.75t^2 + 8.75t + 1.5 = 0$$

2 (seconds)

**A1**

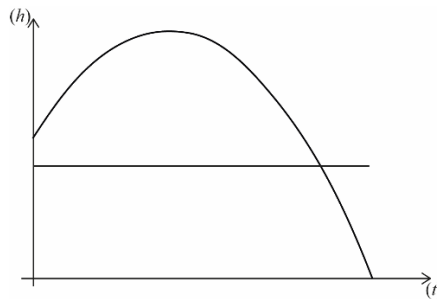
**Note:** If both roots are given, with or without working, award **(M1)A0**.

**[2 marks]**

(c) **METHOD 1**

correct sketch of quadratic function and a straight line in approximate correct position

**(M1)**



1.88 (seconds) (1.87577...(seconds))

**A1**

**METHOD 2**

setting the equation equal to 1.2

**(M1)**

$$-4.75t^2 + 8.75t + 1.5 = 1.2$$

1.88 (seconds) (1.87577...(seconds))

**A1**

**Note:** Award **(M1)A0** if -0.0336702... seen as (part of) a final answer.  
Award **M1A0** for answer of 1.9 seen without working.

**[2 marks]**

(d) Award **R1** for each sensible reason, in the context of the question:

**R1R1**

e.g.

The model ignores air resistance (or wind)

The model treats the ball as a point

The model assumes gravity is constant

The model assumes that the ball continues to follow the trajectory even after hitting the ground

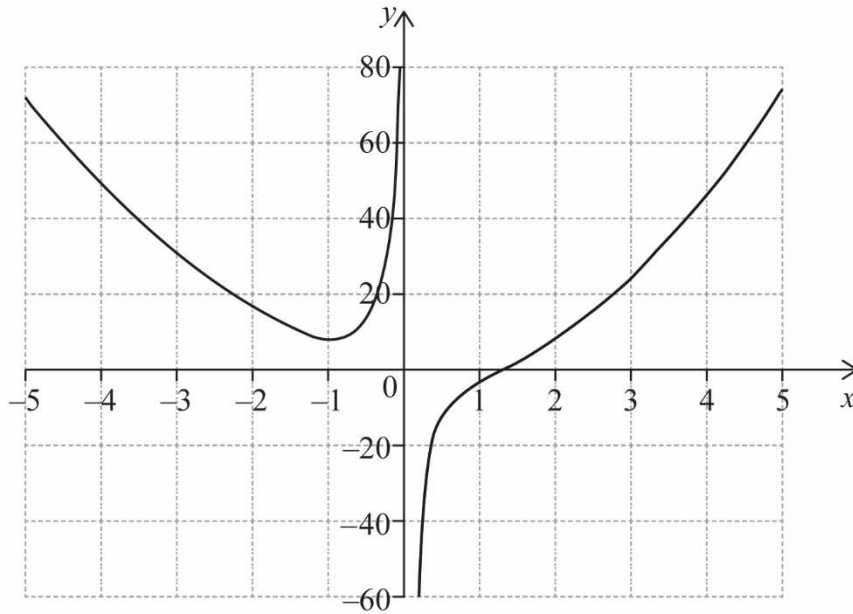
This model ignores the bouncing back of the ball after hitting the ground

**Note:** Do not accept generic criticisms of any mathematical model, such as:  
There are assumptions being made  
Models are never accurate / It is only a model

**[2 mark]**

**Total [8 marks]**

11. (a) (i)



**A1A1A1**

**Note:** Award **A1** for correct shape (curve must be smooth) in the second quadrant only, **A1** for asymptotic behaviour, **A1** for minimum point in approximately correct position AND left-most point around  $(-5, 76)$  (allow a tolerance of half a square for these two points).

(ii)  $-0.941$  ( $-0.941035\dots$ )

**A1**

**[4 marks]**

(b)  $-2.45$  ( $-2.44651\dots$ ),  $-0.252$  ( $-0.252412$ ),  $2.70$  ( $2.69892\dots$ )

**A2**

**Note:** Award **A2** for all three correct, **A1** for two correct and **A0** otherwise. Award at most **A1** if additional solutions are seen. Award **A1A0** for a final answer given as coordinates  $(-2.45, 20)$ ,  $(-0.252, 20)$ ,  $(2.70, 20)$  or  $(2.7, 20)$ . Award **A1A0** for three correct answers given in 2 sf.

**[2 marks]**

(c)  $x = 0$

**A1**

**[1 mark]**

**Total [7 marks]**

12. (a)  $0.15 + 0.2 + k + 0.16 + 2k + 0.25 = 1$  (M1)  
 $k = 0.08$  A1  
 [2 marks]

(b)  $(-4 \times 0.15) + (-3 \times 0.2) + (-1 \times 0.08) + (0 \times 0.16) + (1 \times 0.16) + (4 \times 0.25)$  (M1)  
 $= -0.12$  A1  
 $E(X) \neq 0$  therefore the game is not fair R1

**Note:** Do not award **A0R1** without an explicit value for  $E(X)$  seen. The **R1** can be awarded for comparing **their**  $E(X)$  to zero provided working is shown.

[3 marks]  
 Total [5 marks]

13. (a) attempt at using the trapezoidal rule (M1)  
 $\text{area} = \frac{1}{2}(3 + 2(8 + 19) + 42)$   
 $= 49.5 \text{ (m}^2\text{)}$  A1  
 [2 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) (M1)  
 $\int 3x^2 + 4 \text{ dx} = x^3 + 4x + c$  (A1)(A1)

**Note:** Award **A1** for each correct term.

$f(x) = x^3 + 4x + 3$  A1

**Note:** Award **A1** for simplified correct answer including the value of  $c$ . Accept a value of  $c$  of 3.005 or 3.025 or 2.975 for using the non-integer  $x$ -values and their corresponding  $y$ -values.

[4 marks]

(c) **METHOD 1**  
 forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle (M1)

$\int_0^3 x^3 + 4x + 3 \text{ dx} (= 47.25) + 42 \times 1 + \frac{1}{2} \times 2 \times 42 (= 84)$  (A1)  
 $= 131 \text{ (m}^2\text{)}$  (131.25) A1

**METHOD 2**  
 forming expression for sum of integral and trapezoid (M1)

$\int_0^3 x^3 + 4x + 3 \text{ dx} (= 47.25) + \frac{1}{2} \times 4 \times 42 (= 84)$  (A1)  
 $= 131 \text{ (m}^2\text{)}$  (131.25) A1

**Note:** Award **(A1)** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]  
 Total [9 marks]